PART A - PHYSICS

ALL THE GRAPHS / DIAGRAMS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :

(1) 2.5% (2) 3.5%
(3) 4.5% (4) 6%

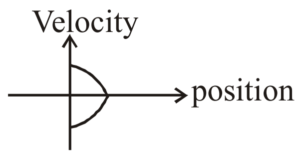
Solution: (3)

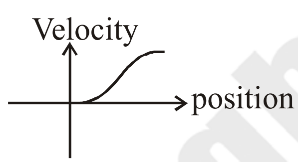
$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\ell^3}$$

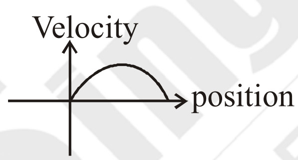
$$\frac{\Delta\rho}{\rho}\times 100 = \frac{\Delta M}{M}\times 100 + 3\frac{\Delta\ell}{\ell}\times 100$$

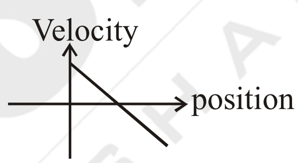
$= 1.5 + 3(1)$
$= 4.5\ \%$

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

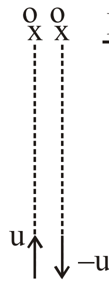
(1) 

(2) 

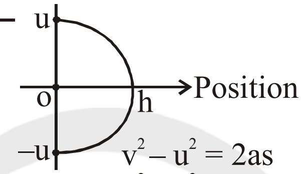
(3) 

(4) 

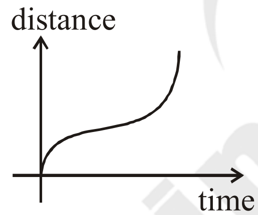
Solution: (2)
Motion is stone thrown vertically up



Position	Velocity
o	+u
h	o
o	–u



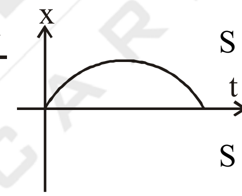
$v^2 - u^2 = 2as$
$v^2 - u^2 = -2gs$



$S = ut + \frac{1}{2}at^2$

Time	Distance
o	o
T	h
2T	2h

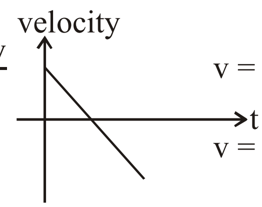
Position	Velocity
o	o
T	h
2T	–u



$S = ut + \frac{1}{2}at^2$

$S = ut - \frac{1}{2}gt^2$

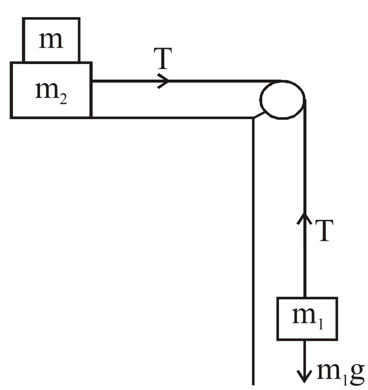
Time	Velocity
o	u
T	o
2T	–u



$v = u + at$

$v = u - gt$

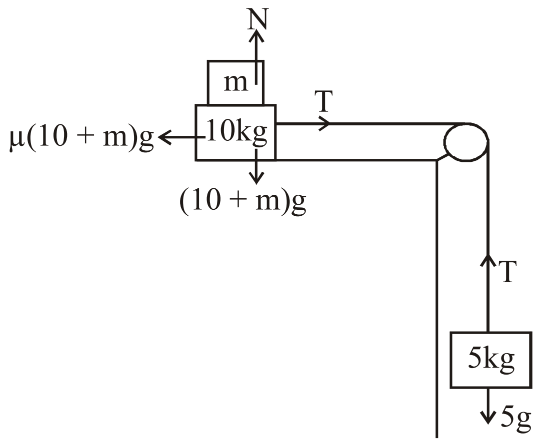
3. Two masses $m_1 = 5\text{kg}$ and $m_2 = 10\text{kg}$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The weight m that should be put on top of m_2 to stop the motion is :



(1) 18.3 kg (2) 27.3 kg

(3) 43.3 kg (4) 10.3 kg

Solution: (2)



system at rest

$\Rightarrow \quad T = \mu(10+m)g \qquad ...(1)$

$T = 5g \qquad ...(2)$

for (1) & (2)

$\mu(10+m)g = 5g$

$0.15(10+m) = 5$

$1.5 + 0.15m = 5$

$0.15 = 3.5m = 3.5$

$\Rightarrow \quad m = \dfrac{3.5}{0.15} = 23.3\text{kg}$

4. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\dfrac{k}{2r^2}$. Its total energy is

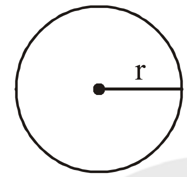
(1) $-\dfrac{k}{4a^2}$ (2) $\dfrac{k}{2a^2}$

(3) Zero (4) $-\dfrac{3}{2}\dfrac{k}{a^2}$

Solution: (3)

Potential energy $U = \dfrac{-k}{2r^2}$

Force $F = -\dfrac{\partial U}{\partial r} = -\dfrac{\partial}{\partial r}\left(\dfrac{-k}{2r^2}\right) = \dfrac{k}{r^3}$



In circular path required centripetal force $= \dfrac{mV^2}{r}$

$$\frac{k}{r^3} = \frac{mV^2}{r}$$

$$\Rightarrow \quad \frac{1}{2}mV^2 = \frac{k}{2r^2}$$

$$\Rightarrow \quad KE = \frac{k}{2r^2}$$

Total energy = KE + PE

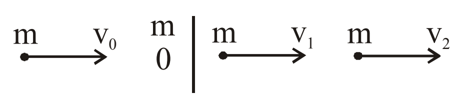
$$= \frac{k}{2r^2} - \frac{k}{2r^2} = 0$$

5. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is :

(1) $\dfrac{v_0}{4}$ (2) $\sqrt{2}\,v_0$

(3) $\dfrac{v_0}{2}$ (4) $\dfrac{v_0}{\sqrt{2}}$

Solution: (2)



Given

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 1.5 \times \frac{1}{2}mv_0^2$$

$\Rightarrow \quad v_1^2 + v_2^2 = 1.5v_0^2 \qquad ...(1)$

From conservation of linear momentum

$P_i = P_f$

$mv_0 + 0 = mv_1 + mv_2$

$v_2 + v_1 = v_0 \qquad ...(2)$

Now $(v_2 - v_1)^2 = 2\left(v_1^2 + v_2^2\right) - (v_2 - v_1)^2$

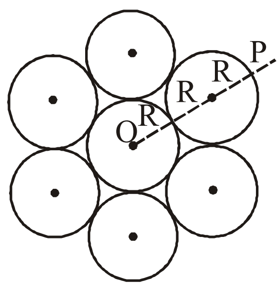
$$= 2\left(1.5v_0^2\right) - v_0^2$$

$$= 3v_0^2 - v_0^2$$

$(v_2 - v_1)^2 = 2v_0^2$

$v_2 - v_1 = \sqrt{2}v_0$

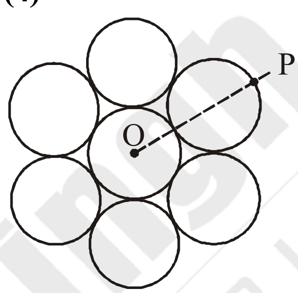
6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is :



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$

(3) $\frac{77}{2}MR^2$ (4) $\frac{181}{2}MR^2$

Solution: (4)



$$I_o = \frac{1}{2}MR^2 + 6\times\left[\frac{1}{2}MR^2 + M(2R)^2\right]$$

$$I_o = \frac{1}{2}MR^2 + 27MR^2$$

$$I_o = \frac{55}{2}MR^2$$

Using theorem of parallel axis

$$I_P = I_o + 7M(3R)^2$$

$$= \frac{55}{2}MR^2 + 63MR^2$$

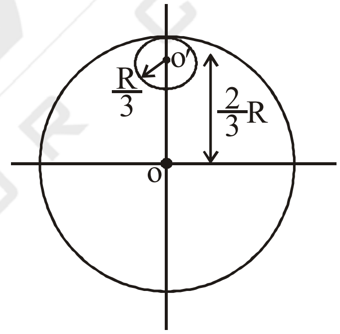
$$= \frac{181}{2}MR^2$$

7. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :

Solution: (1)

MOI of original disc about O $= \frac{1}{2}\times 9M\times R^2$

$$I = \frac{9}{2}MR^2$$



Mass of removed disc $= \sigma\ \pi\left(\frac{R}{3}\right)^2$

$$= \frac{9M}{\pi R^2}\times\frac{\pi R^2}{9} = M$$

MOI of removed disc about O

$$I_{o'} = \frac{1}{2}\times M\times\left(\frac{R}{3}\right)^2 = \frac{1}{18}MR^2$$

MOI of removed disc about O

$$I_o = I_{o'} + M\left(\frac{2}{3}R\right)^2$$

$$= \frac{1}{18}MR^2 + \frac{4}{9}MR^2$$

$$= \frac{9}{18} MR^2 = \frac{1}{2} MR^2$$

MOI of left out disc about O = I – I_o

$$= \frac{9}{2} MR^2 - \frac{1}{2} MR^2 = 4MR^2$$

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then :

(1) $T \propto R^{3/2}$ for any n

(2) $T \propto R^{\frac{n}{2}+1}$

(3) $T \propto R^{(n+1)/2}$

(4) $T \propto R^{n/2}$

Solution: (3)

$$F \propto \frac{1}{R^n}$$

$$F = \frac{k}{R^n}$$

$$m\omega^2 R = \frac{k}{R^n}$$

$$\frac{4\pi^2}{T^2} = \frac{k}{mR^{n+1}} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$\Rightarrow \quad T^2 \propto R^{n+1}$$

$$T \propto R^{\frac{n+1}{2}}$$

9. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is :

(1) $\frac{Ka}{mg}$ (2) $\frac{Ka}{3mg}$

(3) $\frac{mg}{3Ka}$ (4) $\frac{mg}{Ka}$

Solution: (3)

We know that Bulk's Modulus of Elasticity is

$$K = \frac{\Delta P}{-\frac{\Delta V}{V}}$$

$$\frac{\Delta V}{V} = \frac{-\Delta P}{K} \qquad ...(1)$$

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\frac{\Delta r}{r} \qquad ...(2)$$

for (1) and (2)

$$3\frac{\Delta r}{r} = -\frac{\Delta P}{K}$$

$$3\frac{\Delta r}{r} = -\frac{mg}{aK}$$

$$\frac{\Delta r}{r} = -\frac{mg}{3aK}$$

10. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K (b) 2.7 kJ
(2) (a) 195 K (b) –2.7 kJ
(3) (a) 189 K (b) –2.7 kJ
(4) (a) 195 K (b) 2.7 kJ

Solution: (3)

$$PV^\gamma = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}$$

$$\gamma = \frac{5}{3} \text{ [For Monoatomic Gas]}$$

$$TV^{5/3-1} = \text{constant}$$

$TV^{2/3} = \text{constant}$

$T_1V_1^{2/3} = T_2V_2^{2/3}$

$T_1 = 27°V = 300K$

$V_1 = V$

$T_2 = ?$

$V_2 = 2V$

$$\left(\frac{V_2}{V_1}\right)^{2/3} = \frac{T_1}{T_2}$$

$$\left(\frac{2V}{V}\right)^{2/3} = \frac{300}{T_2}$$

$$\Rightarrow T_2 = \frac{300}{2^{2/3}} = \frac{300}{1.59} = 189K$$

$\Delta U = nC_V \Delta T$

$$= \frac{nR}{\gamma - 1}\Delta T$$

$$= 2\times\frac{8.31}{\frac{5}{3}-1}\times(-111)$$

$$= \frac{-2\times 8.31}{\frac{2}{3}}\times(111)$$

$= -2767$

~ -2.7 KJ

11. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly :

(1) $2.35\times10^3\ N/m^2$

(2) $4.70\times10^3\ N/m^2$

(3) $2.35\times10^2\ N/m^2$

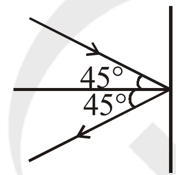
(4) $4.70\times10^2\ N/m^2$

Solution: (1)

$m = 3.32\times10^{-27}$ kg

no. of molecule striking/sec. $n = 10^{23}$

$v = 10^3$ m / s



Total mass hiting/sec = n × m

$= 10^{23}\times3.32\times10^{-27}$

$= 3.32\times10^{-4}$ kg

Force = change in momentum/sec

= 2mv cos 45

$$= 2\times3.32\times10^{-4}\times10^3\times\frac{1}{\sqrt{2}}$$

= 0.664 × 0.707 = 0.4694 N

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{0.4694}{2\times10^{-4}} = 2347N$$

$\sim 2.35\times10^3 N$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver = 108 and Avagadro number = 6.02 × 10^{23} gm $mole^{-1}$)

(1) 6.4 N/m (2) 7.1 N/m

(3) 2.2 N/m (4) 5.5 N/m

Solution: (2)

$\nu = 10^{12}$ / sec

$$m = \frac{M}{N}$$

$$\text{Mass of Atom} = \frac{\text{Atomic Mass}}{\text{Avagadro's Number}}$$

$$m = \frac{108 \times 10^{-3}}{6.02 \times 10^{23}}$$

$$m = 1.79 \times 10^{-25} kg$$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\nu^2 = \frac{k}{4\pi^2 m}$$

$$\Rightarrow \quad k = 4\pi^2\nu^2 m$$

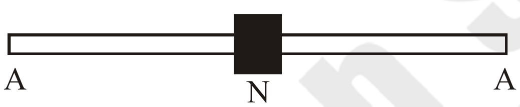
$$k = 4 \times \left(\frac{22}{7}\right)^2 \times \left(10^{12}\right)^2 \times 1.79 \times 10^{-25}$$

$$k = 7.16 N/m$$

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ?

(1) 5 kHz (2) 2.5 kHz
(3) 10 kHz (4) 7.5 kHz

Solution: (1)



$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$

$$\nu = \frac{v}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{Y}{\rho}} = \frac{1}{2 \times 0.6}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$$

$$\nu = 4883 Hz \sim 5 kHz$$

14. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is :

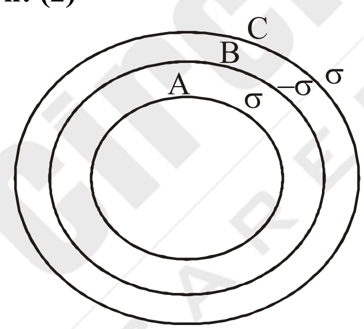
(1) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$

(2) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$

(3) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$

(4) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$

Solution: (2)



$$V_{inside} = \frac{1}{4\pi \epsilon_0}\frac{q}{R} = \frac{\sigma R}{\epsilon_0}$$

$$V_{out} = \frac{1}{4\pi \epsilon_0}\frac{q}{r} = \frac{\sigma R^2}{\epsilon_0 r}$$

$$V_B = V_{BA} + V_{BB} + V_{BC}$$

$$= \frac{\sigma a^2}{\epsilon_0 b} - \frac{\sigma b}{\epsilon_0} + \frac{\sigma c}{\epsilon_0}$$

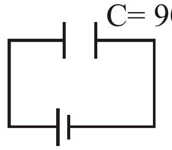
$$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b} - b + c\right)$$

$$= \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

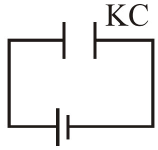
15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :

(1) 1.2 n C (2) 0.3 n C
(3) 2.4 n C (4) 0.9 n C

Solution: (1)



$q = CV$



$q' = KCV$

$$q' = \frac{5}{3} \times 90 \times 10^{-12} \times 20$$

$$q' = 3000 \times 10^{-12} = 3 \times 10^{-9} C$$

$$q' = 3nC$$

Induced charge $= q'\left(1 - \frac{1}{K}\right)$

$$= 3nC\left(1 - \frac{1}{5/3}\right)$$

$$= 3nC\left[1 - \frac{3}{5}\right]$$

$$= 3nC \times \frac{2}{5}$$

= 1.2 nC

16. In an a.c. circuit, the instantaneous e.m.f. and current are given by

e = 100 sin 30 t

$$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively :

(1) 50, 10 (2) $\frac{1000}{\sqrt{2}}, 10$

(3) $\frac{50}{\sqrt{2}}, 0$ (4) 50, 0

Solution: (2)

$$e = 100 \sin 30t$$

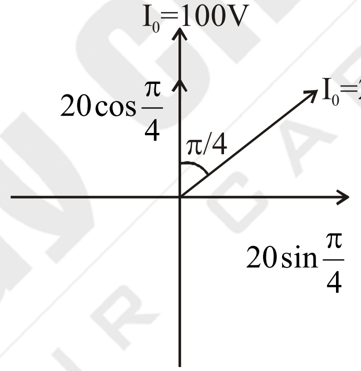
$$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$$

$$\overline{P} = V_r I_r \cos\theta$$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \cos\frac{\pi}{4}$$

$$= \frac{2000}{2} \times \frac{1}{\sqrt{2}}$$

$$\overline{P} = \frac{1000}{\sqrt{2}} \text{ watt}$$



peak value of wattless component of current is

$$= 20 \sin\frac{\pi}{4} = \frac{20}{\sqrt{2}} = 10\sqrt{2} A$$

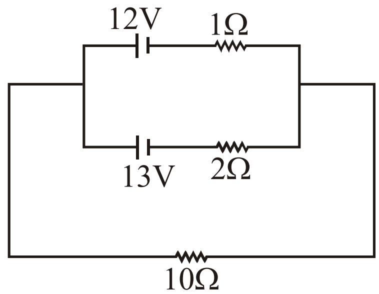
rms value of wattless component of current is

$$= \frac{10\sqrt{2}}{\sqrt{2}} = 10A$$

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between :

(1) 11.6 V and 11.7 V
(2) 11.5 V and 11.6 V
(3) 11.4 V and 11.5 V
(4) 11.7 V and 11.8 V

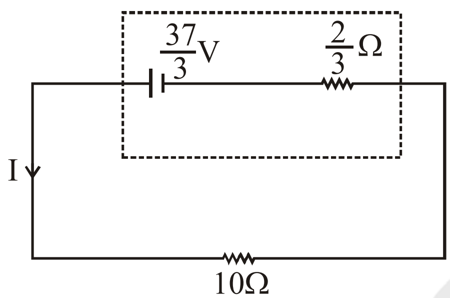
Solution: (2)



$$r_{eq} = \frac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}} = \frac{\frac{37}{2}}{\frac{3}{2}} = \frac{37}{3}\,V$$

$$\frac{1}{r_{eq}} = \frac{1}{1}+\frac{1}{2} = \frac{3}{2}$$

$$\Rightarrow \quad r_{eq} = \frac{2}{3}\,\Omega$$



$$I = \frac{\frac{37}{3}}{10+\frac{2}{3}} = \frac{\frac{37}{3}}{\frac{32}{3}} = \frac{37}{32}\,A$$

$$V_{10\Omega} = IR = \frac{37}{32}\times 10 = 11.56\ \text{volt}$$

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B.

The relation between r_e, r_p, r_α is :

(1) $r_e > r_p = r_\alpha$ (2) $r_e < r_p = r_\alpha$

(3) $r_e < r_p < r_\alpha$ (4) $r_e < r_\alpha < r_p$

Solution: (2)

$$r = \frac{mv}{qB} = \frac{P}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$r \propto \frac{\sqrt{m}}{q}$$

$$r_e : r_p : r_\alpha : \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} : \frac{\sqrt{m_\alpha}}{2e}$$

$$:: \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m}}{e} : \frac{\sqrt{4m}}{2e}$$

$$:: \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m}}{e} : \frac{\sqrt{m}}{e}$$

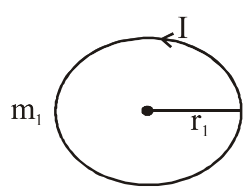
$$r_e < r_p = r_\alpha$$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is :

(1) 2 (2) $\sqrt{3}$

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

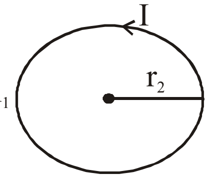
Solution: (3)



$$m_1 = I\pi r_1^2$$

$$B_1 = \frac{\mu_0 I}{2r_1}$$

$m_2 = 2m_1$



$$m_2 = 2m_1$$

$$I\pi r_2^2 = 2\times I\pi r_1^2$$

$$\Rightarrow \quad r_2 = \sqrt{2}\, r_1$$

$$B_2 = \frac{\mu_0 I}{2r_2} = \frac{\mu_0 I}{2\sqrt{2}r_1} = \frac{B_1}{\sqrt{2}}$$

$$\frac{B_1}{B_2} = \frac{\sqrt{2}}{1}$$

20. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_o = \frac{1}{\sqrt{LC}}$ the current exibits resonance. The quality factor, Q is given by :

(1) $\frac{\omega_o L}{R}$ (2) $\frac{\omega_o R}{L}$

(3) $\frac{R}{(\omega_o C)}$ (4) $\frac{CR}{\omega_o}$

Solution: (1)

$$Q = \frac{\omega_0 L}{R}$$

21. An EM wave from air enters a medium. The electric fields are

$\vec{E}_1 = E_{01}\,\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and

$\vec{E}_2 = E_{02}\,\hat{x}\cos\left[k(2z - ct)\right]$ in medium, where the wave number k and frequency *v* refer to their values in air. The medium is non-magnetic. If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct ?

(1) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ (2) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$

(3) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$ (4) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$

Solution: (3)

$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air

$= E_{01}\hat{x}\cos\left[\frac{2\pi\nu}{c}(z - ct)\right]$ in air

$\vec{E}_1 = E_{01}\hat{x}\cos\left[k(z - ct)\right]$ in air

$$\left[\because \frac{2\pi\nu}{c} = \frac{\omega}{c} = k\right]$$

$\vec{E}_k = E_{02}\hat{x}\cos\left[k(2z - ct)\right]$ in medium

Velocity of EM wave in air $= \frac{kc}{k} = c$

Velocity of EM wave of medium $= \frac{kc}{2k} = \frac{c}{2}$

we know that velocity of EM wave is $= \frac{1}{\sqrt{\mu \in}}$

$$\frac{\frac{1}{\sqrt{\in_1}}}{\frac{1}{\sqrt{\in_2}}} = \frac{c}{\frac{c}{2}} = 2$$

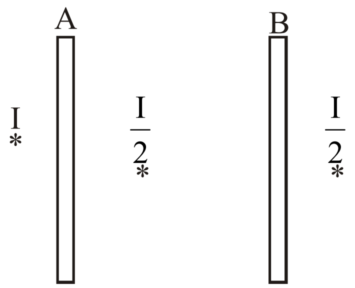
$$\Rightarrow \quad \frac{\in_2}{\in_1} = 4$$

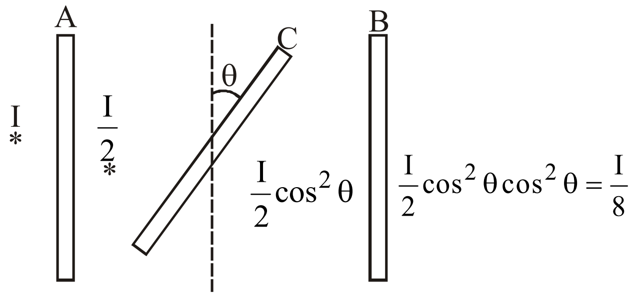
$$\Rightarrow \quad \frac{\in_1}{\in_2} = \frac{1}{4}$$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is :

(1) 0° (2) 30°
(3) 45° (4) 60°

Solution: (3)





$$\cos^4\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

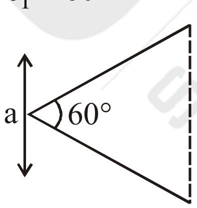
23. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ? (i.e. distance between the centres of each slit.)

(1) 25 μm (2) 50 μm
(3) 75 μm (4) 100 μm

Solution: (1)

$$a = 1\mu m$$

$$\theta_1 = 60°$$



$$\theta = \frac{\theta_1}{2} = 30°$$

$$a\sin\theta = \lambda$$

$$10^{-6}\sin 30 = \lambda \Rightarrow \lambda = 0.5\times10^{-6}\,m$$

YDSE

$$\beta = \frac{\lambda D}{d}$$

$$d = \frac{\lambda D}{\beta} = \frac{0.5\times10^{-6}\times0.5}{10^{-2}}$$

$$d = 25\mu m$$

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

(2) $\Lambda_n \approx A + B\lambda_n$

(3) $\Lambda_n^2 \approx A + B\lambda_n^2$

(4) $\Lambda_n^2 \approx \lambda$

Solution: (1)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow \quad K = \frac{h^2}{2m\lambda^2}$$

$$K_n = \frac{h^2}{2m\lambda_n^2}, K_g = \frac{h^2}{2m\lambda_g^2}$$

$$K_g - K_n = \frac{h^2}{2m\lambda_g^2} - \frac{h^2}{2m\lambda_n^2}$$

$$K_g - K_n = \frac{h^2}{2m}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right) \quad ...(1)$$

We know that

$$E_n = -K_n$$

For emitted photon

$$h\nu = E_n - E_g = K_g - K_n$$

$$\frac{hc}{\Lambda_n} = K_g - K_n$$

$$\Lambda_n = \frac{hc}{K_g - K_n}$$

$$\Lambda_n = \frac{hc}{\frac{h^2}{2m}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right)}$$

$$= \frac{2mc}{h\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2 \lambda_n^2}\right)}$$

$$= \frac{2mc}{h}\frac{\left(\lambda_g^2 \lambda_n^2\right)}{\left(\lambda_n^2 - \lambda_g^2\right)}$$

$$= \frac{2mc}{h}\frac{\lambda_g^2}{\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)}$$

$$\Lambda_n = \frac{2mc}{h}\lambda_g^2\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)^{-1}$$

Now $\lambda_n >> \lambda_g \qquad [\because \lambda_n \propto n]$

$$\Lambda_n = \frac{2mc}{h}\lambda_g^2\left[1 + \frac{\lambda_g^2}{\lambda_n^2} + ...\right]$$

$$\Lambda_n = A + \frac{B}{\lambda_n^2}$$

$$A = \frac{2mc\lambda_g^2}{h}$$

$$B = \frac{2mc\lambda_g^4}{h}$$

25. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is :

(1) $25\nu_L$ (2) $16\nu_L$

(3) $\nu_L/16$ (4) $\nu_L/25$

Solution: (4)

For Lyman series

$$\bar{\nu} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad \left[\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}\right]$$

$$\frac{\nu}{c} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\nu = RC\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

when $n \to \infty \qquad \nu_L = RC \qquad ...(1)$

For Pfund

$$\bar{\nu} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$$

$$\frac{\nu}{c} = R\left(\frac{1}{25} - \frac{1}{n^2}\right)$$

$$\nu = RC\left(\frac{1}{25} - \frac{1}{n^2}\right)$$

when $n \to \infty$

$$\nu_P = \frac{RC}{25} \qquad ...(2)$$

for (1) and (2)

$$\frac{\nu_L}{\nu_P} = 25$$

$$\nu_P = \frac{\nu_L}{25}$$

26. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is Pd ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively :

(1) (.89, .28) (2) (.28, .89)
(3) (0, 0) (4) (9, 1)

Solution: (1)

$\overrightarrow{m \quad u}$ $\quad \begin{matrix} 2m \\ 0 \end{matrix}$ $\Big|$ $\overrightarrow{m \quad v_1}$ $\quad \overrightarrow{2m \quad v_2}$

$$v_1 = \left(\frac{m-2m}{m+2m}\right)u + 0 = \frac{-m}{3m}u = \frac{-u}{3}$$

$$K_i = \frac{1}{2}mu^2 \quad K_f = \frac{1}{2}m\left(-\frac{u}{3}\right)^2 = \frac{K_i}{9}$$

$$\text{fraction loss in KE} = \frac{K_i - \frac{K_i}{9}}{K_i} = \frac{8}{9} = 0.89$$

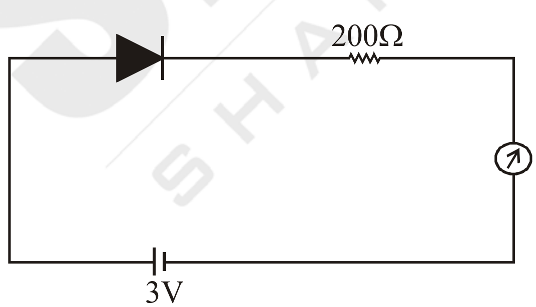
$\overrightarrow{m \quad u}$ $\quad \begin{matrix} 12m \\ 0 \end{matrix}$ $\Big|$ $\overrightarrow{m \quad v_1}$ $\quad \overrightarrow{12m \quad v_2}$

$$v_1 = \left(\frac{m-12m}{m+12m}\right)u = \frac{-11m}{13m}u = \frac{-11}{13}u$$

$$K_i = \frac{1}{2}mu^2, \quad K_f = \frac{1}{2}m\left(\frac{-11}{13}u\right)^2 = \frac{121}{169}K_i$$

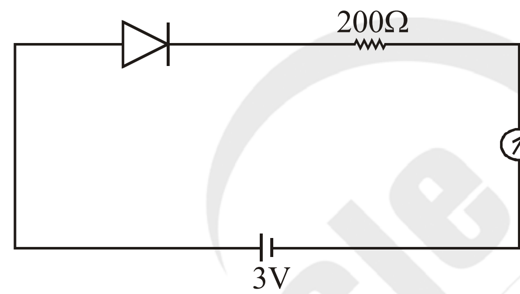
$$\text{fraction loss in KE} = \frac{K_i - \frac{121}{169}K}{K_i} = \frac{48}{169} = 0.28$$

27. The reading of the arnmeter for a silicon diode in the given circuit is :



(1) 0 (2) 15 mA
(3) 11.5 mA (4) 13.5 mA

Solution: (3)



Potential Barrier across Silicon Diode is 0.7 volt

$$I = \frac{V - \Delta V}{R} = \frac{3 - 0.7}{200}$$

$$= \frac{2.3}{200} = 11.5\text{mA}$$

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ?

(1) 2×10^3 (2) 2×10^4
(3) 2×10^5 (4) 2×10^6

Solution: (3)

Since the carrier frequency is distributed as band width frequency, So, 10% of 10 GHz = n × 5 kHz where n = Number of channel

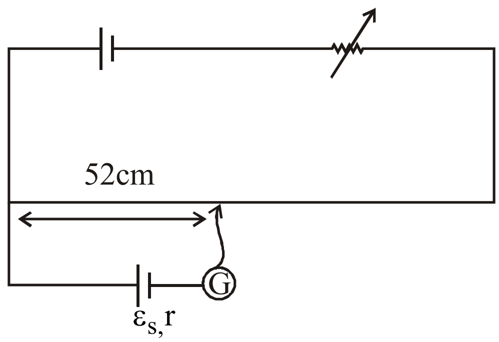
$$\frac{10}{100}\times10\times10^9 = n\times5\times10^3$$

$$n = 2\times10^5 \text{ channel}$$

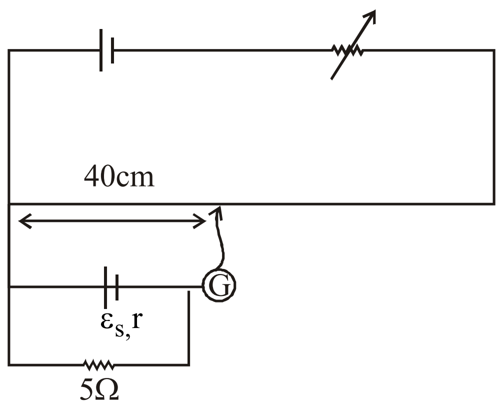
29. ln a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1Ω (2) 1.5Ω
(3) 2Ω (4) 2.5Ω

Solution: (2)



$E_s = 52k, k = \text{potential gradient of wire}$



$V = k\,(40)$

$$\frac{\varepsilon_s}{V} = \frac{k(52)}{k(40)}$$

$$\frac{I(R+r)}{IR} = \frac{13}{10}$$

$$1 + \frac{r}{R} = \frac{13}{10}$$

$$\frac{r}{R} = \frac{13}{10} - 1$$

$$\frac{r}{R} = \frac{3}{10}$$

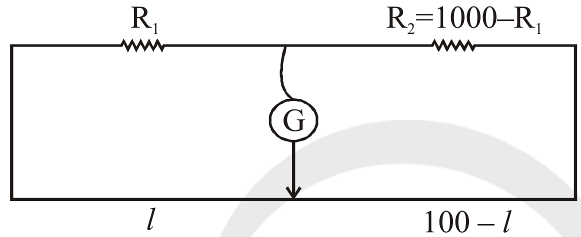
$$\Rightarrow \quad r = \frac{3}{10} \times R = \frac{3}{10} \times 5 = 1.5\Omega$$

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1k Ω. How much was the resistance on the left slot before interchanging the resistances ?

(1) 990 Ω (2) 505 Ω

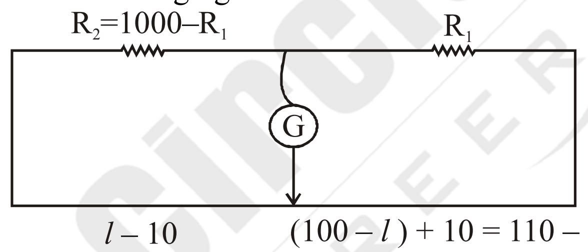
(3) 550 Ω (4) 910 Ω

Solution: (3)



$$\frac{R_1}{R_2} = \frac{\ell}{100-\ell} \qquad ...(1)$$

On Interchanging



$$\frac{R_2}{R_1} = \frac{\ell - 10}{110-\ell}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{(110-\ell)}{(\ell-10)} \qquad ...(2)$$

for (1) and (2) $\quad \dfrac{\ell}{100-\ell} = \dfrac{110-\ell}{\ell-10}$

$$\ell^2 - 10\ell = 11000 - 210\ell + \ell^2$$

$$200\ell = 11000 \Rightarrow \ell = 55\text{cm}$$

for (1) $\quad \dfrac{R_1}{R_2} = \dfrac{\ell}{100-\ell}$

$$\frac{R_1}{1000-R_1} = \frac{\ell}{100-\ell}$$

$$\frac{R_1}{1000-R_1} = \frac{55}{100-55}$$

$$\frac{R_1}{1000-R_1} = \frac{55}{45} = \frac{11}{9}$$

$$9R_1 = 11000 - 11R_1$$

$$20R_1 = 11000$$

$$R_1 = 550\Omega$$

PART B - CHEMISTRY

31. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is:

(1) $C_3H_6O_3$ (2) C_2H_4O
(3) $C_3H_4O_2$ (4) $C_2H_4O_3$

Solution: (4)

$$\frac{\text{Mass of carbon}}{\text{Mass of hydrogen}} = \frac{6}{1}$$

$$\frac{12X}{Y} = 6 \quad | \, 2X = Y \text{ for } C_XH_YO_Z$$

$$C_XH_Y + (X + Y/4)\,O_2 \rightarrow CO_2 + \frac{Y}{2}H_2O$$

No. of oxygen atom required in $C_XH_YO_Z = Z$

No. of oxygen atom required for C_XH_Y combustion

$$= (X + Y/4) \times 2 = 2X + Y/2$$

$$Z = \frac{1}{2}(2X + Y/2)$$

$$Z = X + Y/4 = X + \frac{2X}{4} = \frac{3X}{2}$$

$$X : Y : Z = X : 2X : 3X/2$$
$$= 1 : 2 : 3/2 = 2 : 4 : 3 \text{ or } C_2H_4O_3$$

32. Which type of 'defect' has the presence of cations in the interstitial sites?

(1) Schottky defect
(2) Vacancy defect
(3) Frenkel defect
(4) Metal deficiency defect

Solution: (3)
In Frenkel defect, due to small size of cation it may occupy interstitial space.

33. According to molecular orbital theory which of the following will not be a viable molecule ?

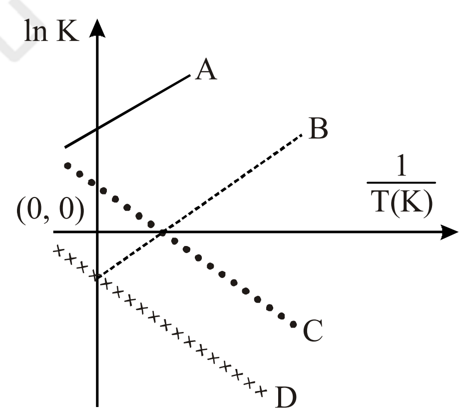
(1) He_2^{2+} (2) He_2^{+}
(3) H_2^{-} (4) H_2^{2-}

Solution: (4)

	M.E.C	B.O.
He_2^{2+}	σ_{1s}^2	1
He_2^{+}	σ_{1s}^1	$\frac{1}{2}$
He_2^{-}	$\sigma_{1s}^2\sigma_{1s}^{*1}$	$\frac{1}{2}$
He_2^{2-}	$\sigma_{1s}^2\sigma_{1s}^{*2}$	0

Does not exist.

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction ?



(1) A and B (2) B and C
(3) C and D (4) A and D

Solution: (1)

$$\ln K = -\frac{\Delta H}{RT} + C$$

If reaction is exothermic, $\Delta H = -ve$,
So slope will be positive.

35. The combustion of benzene (l) gives $CO_2(g)$ and $H_2O(l)$. Given that heat of combustion of benzene at constant volume is – 3263.9 kJ mol^{-1} at 25° C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be :
($R = 8.314\ JK^{-1}\ mol^{-1}$)

(1) 4152.6 (2) –452.46
(3) 3260 (4) –3267.6

Solution: (4)

$$C_6H_6(l) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(l)$$

$$\Delta H = \Delta U + \Delta n_G RT$$

$\Delta H \Rightarrow$ Enthalpy change at constant pressure.

$\Delta U \Rightarrow$ Enthalpy change at constant volume.

$$\Delta n_G = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = -3263.9 \times 10^3 - \frac{3}{2} \times 8.314 \times 298$$

$$= -3267.6\ kJ$$

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point ?

(1) $[Co(H_2O)_6]Cl_3$

(2) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(3) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

(4) $[Co(H_2O)_3Cl_3].3H_2O$

Solution: (1)

$$\Delta T_f = iK_f m$$

For highest freezing point, ΔT_f should be minimum. So 'i' will be minimum.

	i
$[Co(H_2O)_6]Cl_3$	4
$[Co(H_2O)_5Cl]Cl_2.H_2O$	3
$[Co(H_2O)_4Cl_2]Cl.2H_2O$	2
$[Co(H_2O)_3Cl_3].3H_2O$	1

37. An aqueous solution contains 0.10 M H_2S and 0.20M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is:

(1) 5×10^{-8} (2) 3×10^{-20}
(3) 6×10^{-21} (4) 5×10^{-19}

Solution: (2)

$$H_2S \rightleftharpoons H^+ + HS^- \qquad K_1 = 1 \times 10^{-7}$$

$$HS^- \rightleftharpoons H^+ + S^{-2} \qquad K_2 = 1.2 \times 10^{-13}$$

$$\underset{0.1M}{H_2S} + \underset{0.2M}{HCl}$$

$[H^+] = 0.2$ (due to common ion effect.)

$$K_1 = \frac{[H^+][HS^{-1}]}{[H_2S]} = \frac{(0.2)[HS^-]}{0.1} = 10^{-7}$$

$$[HS^-] = 5 \times 10^{-8}$$

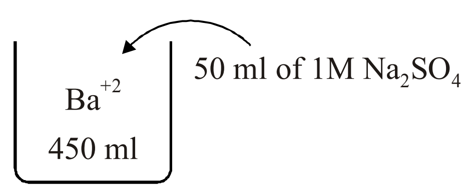
$$K_2 = \frac{[H^+][S^{-2}]}{[HS^-]} = \frac{(0.2)[S^{-2}]}{5 \times 10^{-8}} = 1.2 \times 10^{-13}$$

$$[S^{-2}] = 3 \times 10^{-20}$$

38. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+} ?

(1) 5×10^{-9} M (2) 2×10^{-9} M
(3) 1.1×10^{-9} M (4) 1.0×10^{-10} M

Solution: (3)



$\Rightarrow$ Final volume = 500 ml.

Using dilution formula for $[SO_4^{-2}]$

$$M_1V_1 = M_2V_2$$

$$1 \times 50 = M_2 \times 500$$

$$M_2 = \frac{1}{10}$$

$$[SO_4^{-2}]_{final} = \frac{1}{10}$$

For ppt: $K_{sp} = [Ba^{+2}][SO_4^{-2}]$

$$10^{-10} = [Ba^{+2}] \times \frac{1}{10}$$

$$[Ba^{+2}] = 10^{-9}$$

39. At 518º C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

(1) 2 (2) 3
(3) 1 (4) 0

Solution: (1)

$$CH_3CHO_{(g)} \longrightarrow CH_{4(g)} + CO_{(g)}$$

$$P_o = 363 \text{ torr.}$$

$$r = k\left(P_{CH_3CHO}\right)^n$$

$$1 = k\left(P_o - \frac{5}{100} \times P_o\right)^n \quad .....(i)$$

$$0.5 = k\left(P_o - \frac{33}{100} \times P_o\right)^n \quad .....(ii)$$

Dividing eqn. (i) by (ii)

$$2 = (\sqrt{2})^n, \quad n = 2$$

40. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane?

(Atomic weight of B= 10.8 u)

(1) 6.4 hours (2) 0.8 hours
(3) 3.2 hours (4) 1.6 hours

Solution: (3)

$$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$$

Moles of O_2 required = 3 × 1 = 3

$$\frac{It}{96500} = \text{moles of } O_2 \times 4$$

$$\frac{100 \times t}{96500} = 3 \times 4$$

$$t = \frac{3 \times 4 \times 96500}{100} = 3.2 \text{ hrs.}$$

41. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.Ca(OH)_2]$ to:

(1) $[CaF_2]$

(2) $[3(CaF_2) . Ca(OH)_2]$

(3) $[3Ca_3(PO_4)_2 . CaF_2]$

(4) $[3\{Ca(OH)_2\} . CaF_2]$

Solution: (3)

$$[Ca_3(PO_4)_2.Ca(OH)_2] \xrightarrow{F^-} [Ca_3(PO_4)_3.CaF_2]$$

42. Which of the following compound contain(s) no covalent bond(s) ?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, B_2H_6, PH_3

(2) KCl, H_2SO_4

(3) KCl

(4) KCl, B_2H_6

Solution: (3)

KCl $\rightarrow$ ionic bond.

43. Which of the following are Lewis acid ?

(1) PH_3 and BCl_3 (2) $AlCl_3$ and $SiCl_4$
(3) PH_3 and $SiCl_4$ (4) BCl_3 and $AlCl_3$

Solution: (4)

Lewis acid $\rightarrow$ lone pair acceptor.

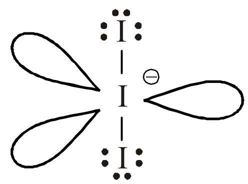
44. Total number of lone pair of electrons in I_3^- ion is:

(1) 3 (2) 6

(3) 9 (4) 12

Solution: (3)

9 lone pair



45. Which of the following salts is the most basic in aqueous solution ?

(1) $Al(CN)_3$ (2) CH_3COOK

(3) $FeCl_3$ (4) $Pb(CH_3COO)_2$

Solution: (2)

$$\underset{}{CH_3COOK} \xrightarrow{H_2O} \underset{\text{weak acid}}{CH_3COOH(aq.)} + \underset{\text{strong base}}{KOH\,(aq.)}$$

46. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively :

(1) $(H_2O + O_2)$ and H_2O

(2) $(H_2O + O_2)$ and $(H_2O + OH^-)$

(3) H_2O and $(H_2O + O_2)$

(4) H_2O and $(H_2O + OH^-)$

Solution: (3)

Oxidation / Reduction:

$$\underset{(+2)}{[Fe\,(CN)6]^{4-}} + \underset{(-1)}{H_2O_2} + 2H^{\oplus} \rightarrow \underset{(+3)}{[Fe\,(CN)6]^{3-}} + \underset{(-2)}{2H_2O}$$

Reduction / Oxidation:

$$\underset{(+3)}{[Fe\,(CN)6]^{3-}} + \underset{(-1)}{H_2O_2} + 2OH^{\ominus} \rightarrow \underset{(+2)}{[Fe\,(CN)6]^{4-}} + \underset{(0)}{O_2} + 2H_2O$$

47. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are:

(1) +3, +4, and +6 (2) +3, +2, and +4

(3) +3, 0, and +6 (4) 3, 0, and +4

Solution: (3)

$[Cr(H_2O)_6]Cl_3$

$x + 6\times 0 + 3\times(-1) = 0$

$x = +3$

$[Cr(C_6H_6)_2]$

Sandwich Comp.

$x + 2 \times 0 = 0$

$x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$2\times(+1) + x + 2\times(-1) + 2(-2) + (-2) + 0 = 0$

$x = +6$

48. The compound that does not produce nitrogen gas by the thermal decomposition is:

(1) $Ba(N_3)_2$ (2) $(NH_4)_2Cr_2O_7$

(3) NH_4NO_2 (4) $(NH_4)_2SO_4$

Solution: (4)

$$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$$

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$$

$$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$$

$$(NH_4)_2SO_4 \xrightarrow{\Delta} \underset{\text{No } N_2 \text{ evolved}}{2NH_3 + H_2SO_4}$$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal'M' is:

(1) Zn (2) Ca

(3) Al (4) Fe

Solution: (3)

$$Al + 3H_2O \xrightarrow{NaOH} \underset{\text{white ppt.}}{Al(OH)_3\downarrow} + \frac{3}{2}H_2$$

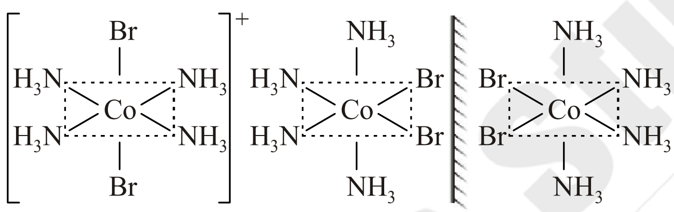
$$\underset{\text{Soluble salt}}{Na[Al(OH)_4]} \xleftarrow{NaOH\,(excess)} ,$$

$$2Al(OH)_3 \xrightarrow{\text{Heat}} \underset{\substack{\text{(Alumina)}\\ \text{used in column}\\ \text{chromatography}}}{Al_2O_3} + 3H_2O$$

50. Consider the following reaction and statements:
$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$
(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.
(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.
(III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.
(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.
The correct statements are :
(1) (I) and (II) (2) (I) and (III)
(3) (III) and (IV) (4) (II) and (IV)

Solution: (2)

$$\underset{ML_4L_2^1\ \text{Type}}{[Co(NH_3)_4Br_2]^+} + Br^- \rightarrow \underset{ML_3L_3^1\ \text{Type}}{[Co(NH_3)_3Br_3]} + NH_3$$



Trans-form
(Optically inactive)

cis-form
(Optically active)
d- and *l*- forms

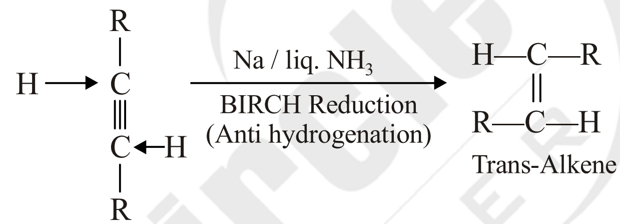
51. Glucose on prolonged heating with HI gives:
(1) Hexane (2) 1-Hexene
(3) Hexanoic acid (4) 6-iodohexanal

Solution: (1)

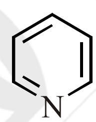
$$\underset{\text{Glucose}}{OHC - (CHOH)_4 - CH_2OH} \xrightarrow[\Delta]{HI} \underset{\text{n-hexane}}{CH_3 - (CH_2)_4 - CH_3}$$

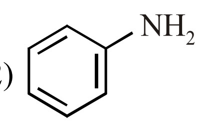
52. The *trans*-alkenes are formed by the reduction of alkynes with :
(1) H_2 - Pd / C, $BaSO_4$
(2) $NaBH_4$
(3) Na / liq. NH_3
(4) Sn - HCl

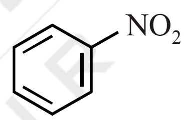
Solution: (3)

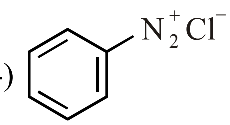


53. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation ?

(1) 

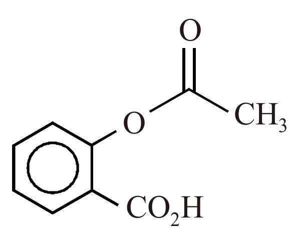
(2) 

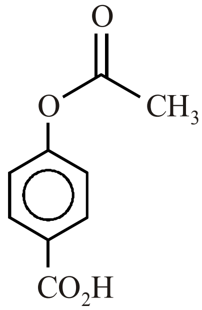
(3) 

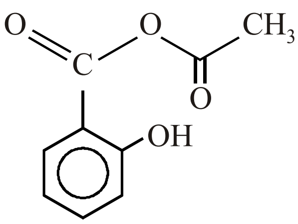
(4) 

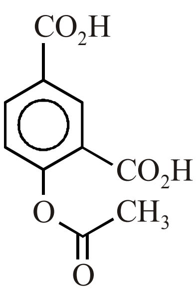
Solution: (2)
The following compounds cannot be estimated for nitrogen by Kjeldahl's method:
(1) Nitrogen containing ring compounds
(2) Nitro compounds
(3) Diazo compounds

54. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound Xas the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

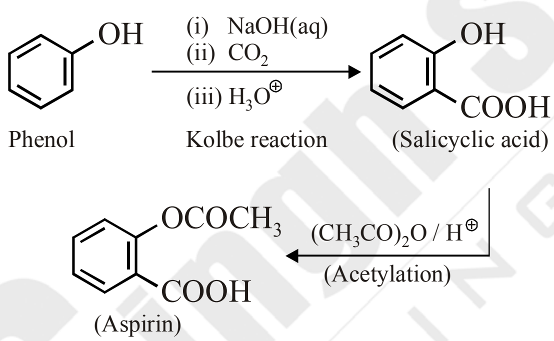
(1) 

(2) 

(3) 

(4) 

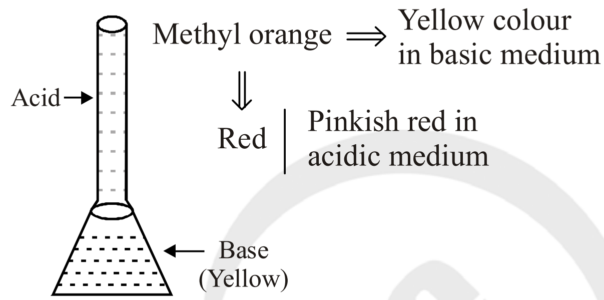
Solution: (1)



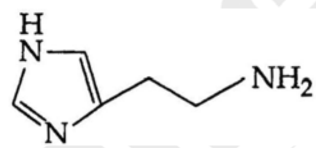
55. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination ?

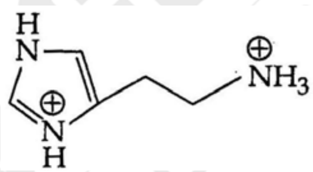
	Base	**Acid**	**End point**
(1)	Weak	Strong	Colourless to pink
(2)	Strong	Strong	Pinkish red to yellow
(3)	Weak	Strong	Yellow to pinkish red
(4)	Stong	Strong	Pink to Colourless

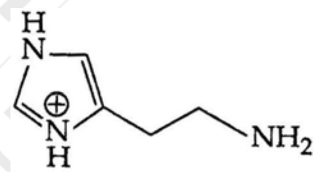
Solution: (3)

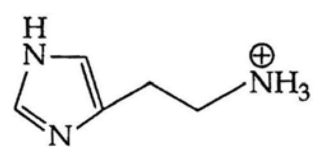


56. The predominant form of histamine present in human blood is (pKa, Histidine = 6.0)

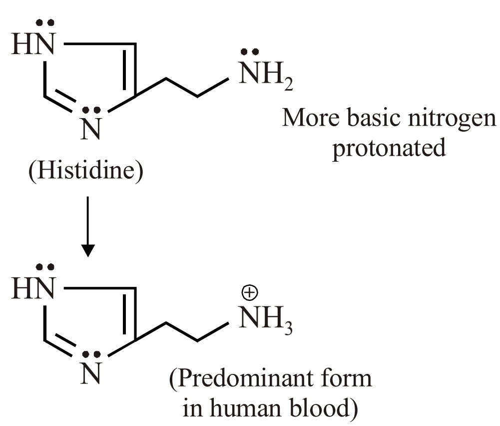
(1) 

(2) 

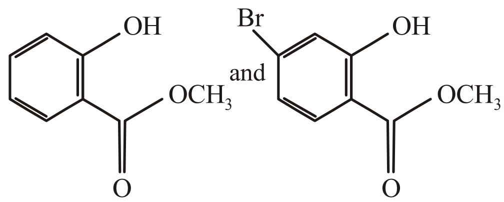
(3) 

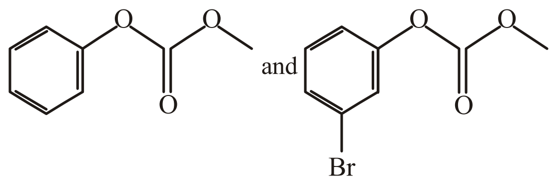
(4) 

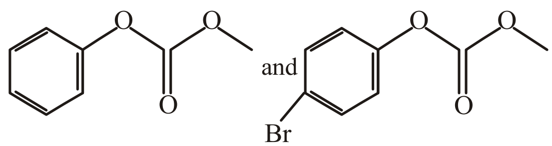
Solution: (4)

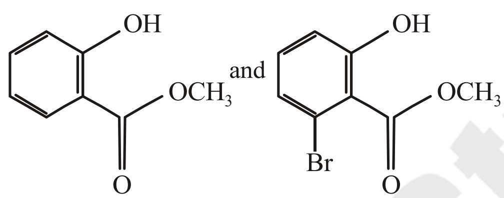


57. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively :

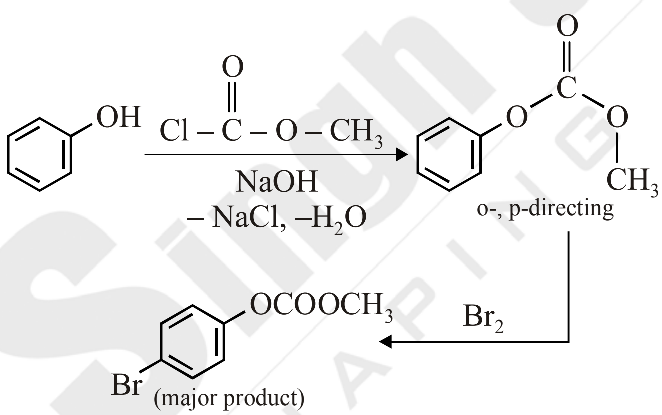
(1) 

(2) 

(3) 

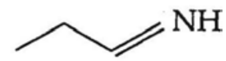
(4) 

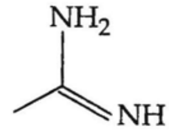
Solution: (3)

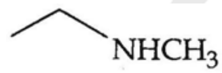


58. The increasing order of basicity of the following compounds is :

(a) 

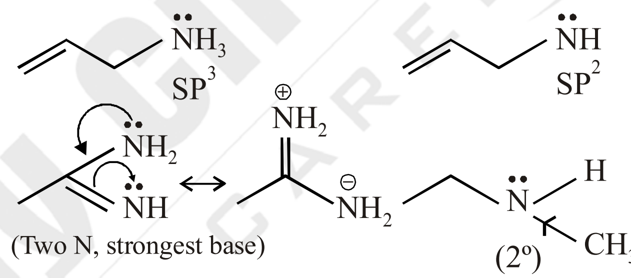
(b) 

(c) 

(d) 

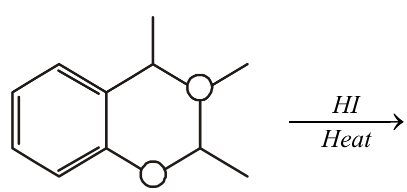
(1) (a) < (b) < (c) < (d)
(2) (b) < (a) < (c) < (d)
(3) (b) < (a) < (d) < (c)
(4) (d) < (b) < (a) < (c)

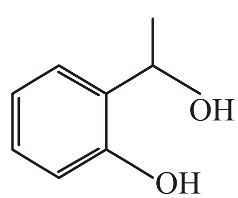
Solution: (3)

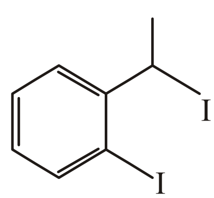


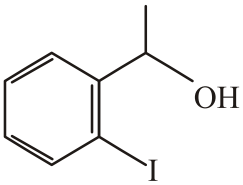
(b) < (a) < (d) < (c)
The combined effect of hybridisation resonance and avilability of lone pair on nitrogen.

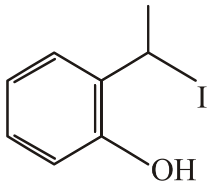
59. The major product formed in the following reaction is:



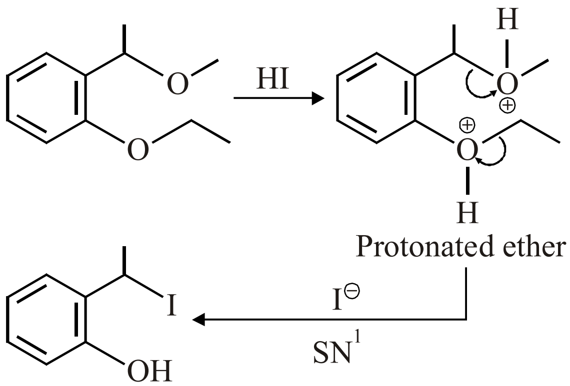
(1) 

(2) 

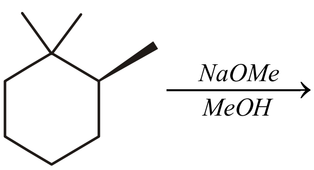
(3) 

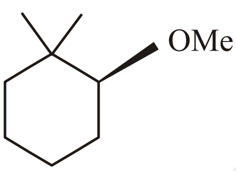
(4) 

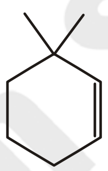
Solution: (4)

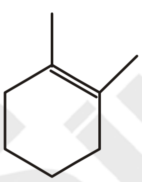
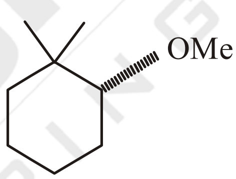


60. The major product of the following reaction is :

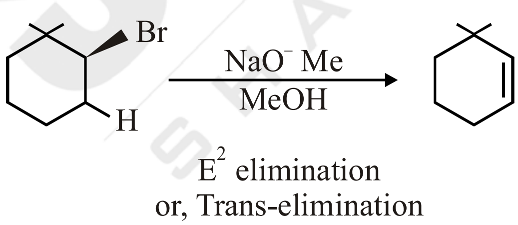


(1) 

(2) 

(3) 
(4) 

Solution: (2)



PART C - MATHEMATICS

61. Two sets A and B are as under :
$A = \{(a, b) \in \mathbf{R} \times \mathbf{R} : |a-5| < 1 \text{ and } |b-5| < 1\}$;
$B = \{(a, b) \in \mathbf{R} \times \mathbf{R} : 4(a-6)^2 + 9(b-5)^2 \le 36\}$.
Then :
(1) $B \subset A$ (2) $A \subset B$
(3) $A \cap B = \phi$ (an empty set)
(4) Neither $A \subset B$ nro $B \subset A$

Solution: (2)

$A = \{(a,b) \in R \times R : |a-5| < 1 \,\&\, |b-5| < 1\}$

$\because |a-5| < 1$

$\Rightarrow -1 < a-5 < 1$

$\Rightarrow 4 < a < 6 \Rightarrow a \in (4,6)$

Also $b \in (4,6)$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}$

$4(a-6)^2 + 9(b-5)^2 \le 36$

$$\Rightarrow \frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \le 1$$

It is an eq. of ellipse having centre (6, 5) & length of semi major & axis = 3
length of semi minor axis = 2

$\Rightarrow 3 < a < 9 \Rightarrow a \in (3,9)$

$\& \; 3 < b < 7 \Rightarrow b \in (3,7)$

Clearly $A \subset B$

62. Let $S = \{x \in R : x \ge 0$ and

$2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\}$

(1) is an empty set.
(2) contains exactly one element.
(3) contains exactly two element.
(4) contains exactly four element.

Solution: (3)

$$2\left|\sqrt{x}-3\right|+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$$

Let $\sqrt{x}=t$

$$2|t-3|+t(t-6)+6=0$$

If t > 3, $\quad 2t-\not{6}+t^2-6t+\not{6}=0$

$$t^2-4t=0$$

$$t(t-4)=0$$

$t=0, t=4,$ t = 0 does not existing t > 3

∴ t = 4

$$\sqrt{x}=4 \Rightarrow x=16$$

If t < 3 $\quad -2(t-3)+t(t-6)+6=0$

$$-2t+6+t^2-6t+6=0$$

$$t^2-8t+12=0$$

$$(t-6)(t-2)=0$$

t = 6, $\quad$ t = 2 → exist

$$\sqrt{x}=2 \Rightarrow x=4$$

Two Solution Exist (16, 4)

63. If α, β, ∈ C are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

(1) –1 (2) 0

(3) 1 (4) 2

Solution: (3)

$$x^2-x+1=0$$

$$x=\frac{-(-1)\pm\sqrt{-3}}{2}$$

$$x=\frac{1\pm\sqrt{3}i}{2}$$

We know that $w=\dfrac{-1+\sqrt{3}i}{2}$

Let $\alpha=-w$

$\beta=-w^2 \quad \Rightarrow \alpha^{101}+\beta^{107}$

$$=(-w)^{101}+\left(-w^2\right)^{107}$$

$$=-\left(w^{101}+w^{214}\right)$$

$$=-\left(w^{99}w^2+w^{213}w\right)$$

$$=-\left(w^2+w\right)=-(-1)=1$$

64. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}=(A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to :

(1) (–4, –5) (2) (–4, 3)

(3) (–4, 5) (4) (4, 5)

Solution: (3)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}=(A+Bx)(x-A)^2$$

$$R_1+R_1+R_2+R_3$$

$$\begin{vmatrix} 5x-4 & 5x-4 & 5x-4 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}=(A+Bx)(x-A)^2$$

$$(5x-4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}=(A+Bx)(x-A)^2$$

$$(5x-4)\begin{vmatrix} 0 & 1 & 1 \\ x+4 & x-4 & 2x \\ 0 & 2x & x-4 \end{vmatrix}=(A+Bx)(x-A)^2$$

$$-(5x-4)(x+4)(x-4-2x)=(A+Bx)(x-A)^2$$

$$-(5x-4)(x+4)(-4-x)=(A+Bx)(x-A)^2$$

$$(5x-4)(x+4)^2=(A+Bx)(x-A)^2$$

$$A=-4,\ B=5$$

65. If the system of linear equations

x + ky + 3z = 0

3x + ky – 2z = 0

2x + 4y – 3z = 0

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) – 10 (2) 10

(3) – 30 (4) 30

Solution: (2)

$$x+11y+3z=0 \quad \text{... (i)} \quad x=-11y-3z$$
$$3x+11y-2z=0 \quad \text{... (ii)}$$
$$2x+4y-3z=0 \quad \text{... (iii)}$$
$$3(-11y-3z)+11y-2z=0$$
$$-33y-9z+11y-2z=0$$
$$-2y=11z,\quad z=-2y$$
$$x=-11y-3z$$
$$x=-11y-3(-2y)$$
$$x=-11y+6y$$
$$x=-5y$$
$$\frac{xz}{yz}=\frac{(-5y)(-2y)}{y^2}=10$$

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 1000 (2) less than 500

(3) at least 500 but less than 750

(4) at least 750 but less than 1000

Solution: (1)

To select 4 movels from 6 novels.

no. of selection = ${}^6C_4$

To select 1 dictionary from 3 different dictionaries

no. of selection = ${}^3C_1$

Total no. of selection = ${}^6C_4 \times {}^3C_1$

$$=\frac{6!}{4!21}\times 3$$
$$=15\times 3$$
$$=45$$

Now since dictionary must be in middle

$\therefore$ we have to arrange 4 selected novels.

$\therefore$ total no. of arrangement = 45 × ${}^4C_4$

$$=45\times 4!$$
$$=45\times 24$$
$$=1080$$

67. The sum of the co-efficients of all odd degree terms in the expansion of

$$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5,(x>1) \text{ is :}$$

(1) –1 (2) 0

(3) 1 (4) 2

Solution: (4)

$$(a+b)^5+(a-b)^5=2\left[{}^5C_0a^5+{}^5C_2a^3b^2+{}^5C_4ab^4\right]$$
$$\Rightarrow\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$$
$$=2\left[x^5+10x^3\left(x^3-1\right)+5x.\left(x^3-1\right)^2\right]$$
$$=2\left[x^5+10x^6-10x^3+5x^7-10x^4+5x\right]$$

$\therefore$ coeff. of odd term $=2[1-10+5+5]=2$

68. Let $a_1, a_2, a_3, \ldots\ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{21} a_{4k+1}=416$ and $a_9 + a_{43} = 66$. If

$a\frac{2}{1}+a\frac{2}{2}+....+a\frac{2}{17}=140\text{m}$, then m is equal to :

(1) 66 (2) 68

(3) 34 (4) 33

Solution: (3)

$$\sum_{k=0}^{12} a_{4k+1} = 416$$

$$\Rightarrow a_1 + a_5 + a_9 + ... + a_{45} + a_{49} = 416$$

$$\Rightarrow (a_1 + a_{49}) + (a_5 + a_{45}) + (a_9 + a_{41})$$
$$+ (a_{13} + a_{37}) + (a_{17} + a_{33}) + (a_{21} + a_{29}) + a_{25} = 416$$

$$6[a_1 + a_{49}] + a_{25} = 416$$

$$\Rightarrow a + 24d = 32 \quad ... (i)$$

Also $a_9 + a_{42} = 66$

$$\Rightarrow a + 25d = 33 \quad ... (ii)$$

d = 1   a = 8

$$\Rightarrow a_1^2 + a_2^2 + a_3^2 + ...a_{17}^2 = 140m$$

$$\Rightarrow 8^2 + 9^2 + 10^2 + ...(24)^2 = 140m$$

$$\Rightarrow \left[1^2 + 2^2 + 3^2 + ...(24)^2\right] - \left[1^2 + 2^2 + ...7^2\right] = 140m$$

$$\Rightarrow 4900 - 140 = 140m$$
$$\Rightarrow 4760 = 140m$$
$$m = 34$$

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$

If B – 2A = 100λ, then λ is equal to :

(1) 232   (2) 248
(3) 464   (4) 496

Solution: (2)

S = $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$

$$A = \left[1^2 + 2^2 + 3^2 + ...20^2\right] + \left[2^2 + 4^2 + 6^2 + ...20^2\right]$$

$$A = \left[1^2 + 2^2 + 3^2 + ...20^2\right] + 4\left[1^2 + 2^2 + 3^2 + ...10^2\right]$$

$$A = \frac{20 \times 21 \times 41}{6} + 4 \times \frac{10 \times 11 \times 21}{6}$$

$$A = 2870 + 1540$$
$$A = 4410$$

$$B = \left[1^2 + 2^2 + ...40^2\right] + 4\left[1^2 + 2^2 + ...20^2\right]$$

$$= \frac{40 \times 41 \times 81}{6} + 4 \times \frac{20 \times 21 \times 41}{6}$$

$$= 22140 + 11480$$
$$B = 33620$$

Now B – 2A = 33620 – 8820 = 100λ
= 24800 = 100λ
$\Rightarrow \lambda = 248$

70. For each t ∈ **R**, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$$

(1) is equal to 0.   (2) is equal to 15.
(3) is equal to 120.   (4) does not exist (in **R**).

Solution: (3)

In $x \to 0 + \left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

$$\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \le \frac{1}{x}$$

$$\frac{2}{x} - 1 < \left[\frac{2}{x}\right] \le \frac{2}{x}$$

$$\frac{15}{x} - 1 < \left[\frac{15}{x}\right] \le \frac{15}{x}$$

$$\frac{1}{x}(1 + 2 + ... + 15) - 15 < \left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right] \le \frac{1}{x}(1 + 2 + ... + 15)$$

In $x > 0 + \frac{\cancel{x}}{\cancel{x}}\left(\frac{1}{x}(1 + 2 + ... + 15) - 15x < x + 0 + x\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right] \le x + 0 + \frac{\cancel{x}}{\cancel{x}}(1 + 2 + ...15)\right)$

$$\frac{15(15+1)}{2} - 0 < x \to 0 + x\left[\frac{1}{x}\right] +$$

$$\left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right] \le \frac{15(15+1)}{2}$$

$$x \to 0 + x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right) = 120$$

71. Let S = {t ∈ **R** : f (x) = |x – π|· ($e^{|x|}$ – 1) sin |x| is not differentiable at t}. Then the set S is equal to :

(1) ϕ (an empty set) (2) {0}

(3) {π} (4) {0, π}

Solution: (1)

$$f(x) = |x-\pi| \cdot \left(e^{|x|} - 1\right)\sin|x|$$

$L.H.D$ at $x = 0$

$$\lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h+0} \frac{f(-h) - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{|-h-\pi|\left(e^{|-h|} - 1\right)\sin|-h| - 0}{-h}$$

$$= 0(0+\pi)(1)\times 1$$

$R.H.D$ of $x = 0$ $\quad = 0$

$$\lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \frac{|h-\pi|\left(e^{|h|} - 1\right)\sin|h| - 0}{h}$$

At $x = \pi$ $\quad = 0$

$$= \lim_{x\to 0} \frac{f(\pi - h) - f(\pi)}{h}$$

$$= \lim_{x\to 0} \frac{(\not\pi - h - \not\pi)\left(e^{|\pi - h|} - 1\right)\sin|\pi - h| - 0}{-h}$$

$R.H.D$ of $x = \pi$

$$= \lim_{x\to 0} \frac{f(\pi + h) - f(\pi)}{h}$$

$$= \lim_{x\to 0} \frac{1\not\pi + h - \not\pi 1\left(e^{|\pi + h|} - 1\right)\sin|\pi + h| - 0}{h}$$

$$= \lim_{x\to 0} \frac{h\left(e^{h+\pi} - 1\right)(\sinh)}{h}$$

∴ differentiable at 0 and π = 0

72. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) 6 (2) $\frac{7}{2}$

(3) 4 (4) $\frac{9}{2}$

Solution: (4)

$$y^2 = 6x, \quad 9x^2 + by^2 = 16$$

$$\frac{9x^2}{16} + \frac{by^2}{16} = 1$$

$$2yy^1 = 6$$

$$2ym_1 = 6$$

$$m_1 = \frac{6}{2y_1}$$

$$9(2x_1) + 2by_1y^1 = 0$$

$$18x_1 + 2by_1y^1 = 0$$

$$y_1 = \frac{-18x_1}{2by_1}$$

$$y_1 = \frac{-9x_1}{by_1}$$

$$m_2 = \frac{-9x_1}{by_1}$$

$$m_1m_2 = 1$$

$$\left(\frac{\not 6}{\not 2 y_1}\right)\left(\frac{-9x_1}{by_1}\right) = -1$$

$$by_1^2 = 27x_1$$

$$b(6x_1) = 27x_1 \quad b = \frac{27}{6}$$

$$b = \frac{9}{2}$$

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is :

(1) 3 (2) – 3

(3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

Solution: (4)

$$f(x) = x^2 + \frac{1}{x^2} \text{ and } g(x) = x - \frac{1}{x}$$

$$h(x) = \frac{f(x)}{g(x)} = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \frac{\left(x - \frac{1}{x}\right)^2 + 2}{\left(x - \frac{1}{x}\right)}$$

$$= \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

$$\log t = x - \frac{1}{x}$$

$$h(x) = t + \frac{2}{t}$$

$$h^1(x) = 1 - \frac{2}{t^2}$$

$$= \frac{(t - \sqrt{2})(t + \sqrt{2})}{t^2}$$

Local minima of $x = \sqrt{2}$

$$\therefore \; h(x)\min = \left(x - \frac{1}{x}\right) + \frac{2}{x - \frac{1}{x}}$$

$$= \sqrt{2} + \frac{2}{\sqrt{2}} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

74. The integral

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$$

is equal to :

(1) $\frac{1}{3(1 + \tan^3 x)} + c$

(2) $\frac{-1}{3(1 + \tan^3 x)} + c$

(3) $\frac{1}{1 + \cot^3 x} + c$

(4) $\frac{-1}{1 + \cot^3 x} + c$

(where C is a constant of integration)

Solution: (2)

$$I = \int \frac{\sin^2 x \cos^2 x dx}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2}$$

$$I = \int \frac{\sin^2 x \cos^2 x dx}{\left(\sin^2 x + \left(sin^3 x + \cos^3 x\right) + \cos^2 x\left(\sin^3 x + \cos^3 x\right)\right)^2}$$

$$I = \int \frac{\sin^2 x \cos^2 x dx}{\left(\left(sin^3 x + \cos^3 x\right)\left(\sin^2 x + \cos^2 x\right)\right)^2}$$

$$I = \int \frac{\sin^2 x \cos^2 x dx}{\left(sin^3 x + \cos^3 x\right)^2}$$

$$= \int \frac{\tan^2 x \sec^2 x dx}{\left(\tan^3 x + 1\right)^2}$$

$$t = \tan x, \quad dt = \sec^2 x dx$$

$$= \int \frac{t^2\,dt}{\left(t^3+1\right)^2} = \frac{1}{3}\int \frac{3t^2\,dt}{\left(t^3+1\right)^2}$$

$$= \frac{1}{3}\frac{\left(t^3+1\right)^{-2+1}}{-2+1}$$

$$= \frac{-1}{3\left(t^3+1\right)} = \frac{-1}{3\left(\tan^3 x+1\right)}$$

75. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}dx$ is :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{2}$

(3) 4π (4) $\frac{\pi}{4}$

Solution: (4)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2x}dx \quad \ldots \text{(i)}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2\left(\cancel{\pi/2} - \cancel{\pi/2} - x\right)dx}{1+2\left(\cancel{\pi/2} - \cancel{\pi/2} - x\right)}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\left(-\sin x\right)^2 dx}{1+2^{-x}}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x dx}{2^x+1} \quad \ldots \text{(ii)}$$

(i) + (ii)

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} + \frac{2^x \sin^2 x}{1+2^x}dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x\cancel{\left(2^x+1\right)}}{\cancel{\left(2^x+1\right)}}dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 xdx$$

$$\cancel{2}I = \cancel{2}\int_{0}^{\pi/2} \sin^2 xdx$$

$$I = \int_{0}^{\pi/2} \frac{1-\cos^2 x}{2}$$

$$I = \frac{1}{2}\int_{0}^{\pi/2} \left(1-\cos^2 x\right)dx$$

$$= \frac{1}{2}\left[x - \frac{\sin^2 x}{2}\right]_0^{\pi/2}$$

$$I = \frac{1}{2}\left[\frac{\pi}{2} - 0 - 0\right]\frac{\pi}{4}$$

76. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and α, β, (α < β) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0.$ Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and y = 0, is :

(1) $\frac{1}{2}\left(\sqrt{3}-1\right)$ (2) $\frac{1}{2}\left(\sqrt{3}+1\right)$

(3) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$ (4) $\frac{1}{2}\left(\sqrt{2}-1\right)$

Solution: (1)

$$g(x) = \cos x^2, f(x) = \sqrt{x}$$

$y = gof(x) = gf(x) = g$

$(\sqrt{x}) = \cos(\sqrt{x})^2 = \cos x$

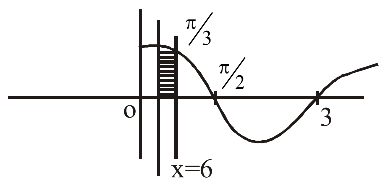
$18x^2 - 9\pi x + \pi^2 = 0$

$18x^2 - 6\pi x + 3\pi x + \pi^2 = 0$

$6x(3x - \pi) - \pi(3x - \pi) = 0$

$x = \pi/6, x = \pi/3$

$\alpha = \pi/6, \beta = \pi/3, \alpha < \beta$



$$\int_{\pi/6}^{\pi/3} \cos x dx = [\sin x]_{\pi/6}^{\pi/3}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$= \frac{1}{2}(\sqrt{3} - 1) \text{sq. unit}$$

77. Let y = y(x) be the solution of the differential equation $\sin x \frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{4}{9\sqrt{3}}\pi^2$ (2) $\frac{-8}{9\sqrt{3}}\pi^2$

(3) $-\frac{8}{9}\pi^2$ (4) $-\frac{4}{9}\pi^2$

Solution: (3)

$$\sin x \frac{dy}{dx} + y\cos x = 4x$$

$$\frac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x$$

$$I.f = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$$

$$y.\sin x = \int \sin x \; 4x \operatorname{cosec} x dx$$

$$y \sin x = \int 4x dx$$

$$y \sin x = 2x^2 + C$$

$$O \sin \pi/2 = 2(\pi/2)^2 + C$$

$$O = \frac{\pi^2}{2} + C$$

Now $C = -\pi^2/2$

$$y \sin x = 2x^2 - \frac{\pi^2}{2}$$

$$y(\pi/6) \sin \pi/6 = 2(\pi/6)^2 - \frac{\pi^2}{2}$$

$$y(\pi/6) \cdot \frac{1}{2} = 2(\pi/36) - \frac{\pi^2}{2}$$

$$y(\pi/6) \cdot \frac{1}{2} = \frac{\pi^2 - 9\pi^2}{18} = \frac{-8\pi^2}{18}$$

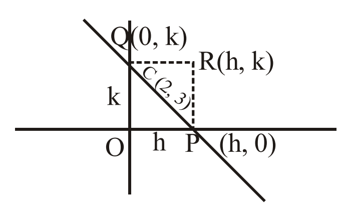
$$y(\pi/6) = \frac{-8}{9}\pi^2$$

78. A straight line through a fixed point (2, 3) intersets the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) 3x + 2y = 6 (2) 2x + 3y = xy

(3) 3x + 2y = xy (4) 3x + 2y = 6xy

Solution: (3)



eqn. of PQ : $\frac{x}{h}+\frac{y}{k}=1$

$\because$ $(2,3)$ *lies on PQ*

$\Rightarrow \frac{2}{h}+\frac{3}{k}=1$

So locus is $\frac{2}{x}+\frac{3}{y}=1$

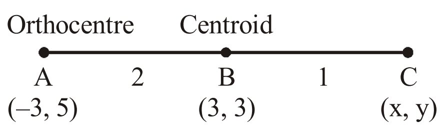
$\Rightarrow 3x+2y=xy$

79. Let the orthocentre and centriod of a triangle be A (–3, 5) and B (3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

(1) $\sqrt{10}$ (2) $2\sqrt{10}$

(3) $3\sqrt{\frac{5}{2}}$ (4) $\frac{3\sqrt{5}}{2}$

Solution: (3)



$\because AB:BC=2:1$

Let C (x, y)

$\Rightarrow \frac{2x-3}{3}=3$ $\frac{2y+5}{3}=3$

$\Rightarrow 2x-3=9$ $2y+5=9$

$2x=12$ $2y=4$

$x=6$ $y=2$

$\therefore$ *Coordinate of* $C(6,2)$

$AC=\sqrt{(6+3)^2+(2-5)^2}$

$=\sqrt{81+9}$

$=\sqrt{90}$

$AC=3\sqrt{10}$

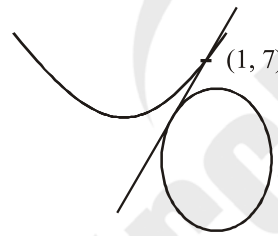
$\Rightarrow r=\frac{3\sqrt{10}}{2}=3\sqrt{\frac{5}{2}}$

80. If the tangent at (1, 7) to the curve x2 = y – 6 touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is :

(1) 195 (2) 185

(3) 85 (4) 95

Solution: (4)



Eqn. of tangent (1, 7) to a curve

$x^2=y-6$

$xx_1=\frac{y+y_1}{2}-6$

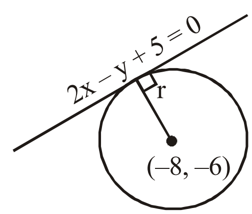
$x=\frac{y+7}{2}-6$

$2x=y+7-12$

$2x-y=-5$

$2x-y+5=0$ is tangent of a curve

$x^2+y^2+16x+12y+c=0$



$r=\left|\frac{2(-8)-(-6)+5}{\sqrt{2^2+(-1)^2}}\right|$

$r=\left|\frac{-16+6+5}{\sqrt{5}}\right|$

$\sqrt{(-8)^2+(-6)^2-C}=\left|\frac{-5}{\sqrt{5}}\right|=\sqrt{5}$

$64+36-C=5$
C = 100 – 5
C = 95

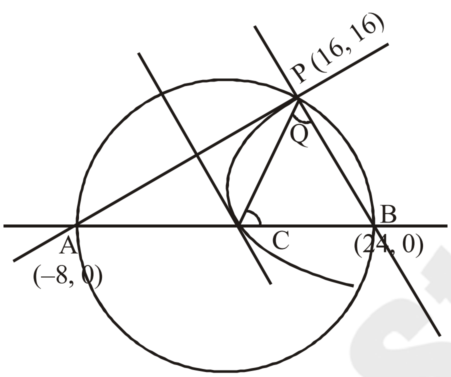
81. Tangent and normal are drawn at P (16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A adn B, respectively. If C is the centre of the circle through the points P, A and B and ∠CPB = θ, then a value of tan θ is :

(1) $\frac{1}{2}$ (2) 2

(3) 3 (4) $\frac{4}{3}$

Solution: (2)

$y^2=16x$



Eqn. of tangent of (16, 16)

$$yy_1=16\left(\frac{x+x_1}{2}\right)$$

$$16y=16\left(\frac{x+16}{2}\right)$$

$2y=x+16 \Rightarrow x-2y+16=0$

put, $y=0, x=-16$

point, $A\ is(-16,0)$ $\frac{-1}{-2}=\frac{1}{2}=m$

point f $A\ is(-16,0)$

Eqn. of normal at (16, 16)

$y-16=-2(x-16)\left\{slope\ of\ \text{tangent}=\frac{1}{2}\right.$

$2x+y=48$

put $y=0, x=24$

point $B\ is(24,0)$

C is the mid-point of AB

C = (4, 0)

Slope of PB = –2

Slope of PC = $\frac{4}{3}$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$=\left|\frac{-2-4/3}{1+(-2)4/3}\right|$$

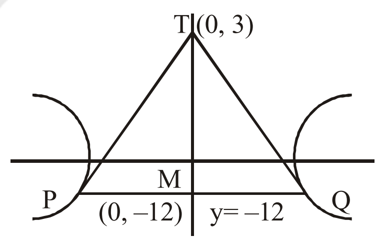
$\tan\theta=2$

82. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $45\sqrt{5}$ (2) $54\sqrt{3}$

(3) $60\sqrt{3}$ (4) $36\sqrt{5}$

Solution: (1)



Equation of chord of contact at T (0, 3)

$$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

$$\Rightarrow \frac{0}{9}-\frac{3y_1}{36}=1$$

$$\Rightarrow y_1=-12$$

Now put $y=-12$ *in eq. of Hyperbola*

$$4x^2-144=36$$

$\Rightarrow 4x^2 = 180$

$\Rightarrow x = \sqrt{45}$

$\therefore \quad PQ = 2x = 6\sqrt{5}$

$= ar(\Delta PTQ) = \frac{1}{2} \times PQ \times TM$

$= \frac{1}{2} \times 6\sqrt{5} \times 15 \; = 45\sqrt{5}$

83. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is :

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Solution: (2)

Plane passing through line of intersectin fo first two planes, is

$= \; :(2x - 2y + 32 - 2) + \lambda(x - y + z + 1) = 0$

$\Rightarrow \pi : x(\lambda + 2) - y(2 + \lambda) + z(\lambda + 3) + (\lambda - 2) = 0$

Plane π has infinite no. of solution

with $x + 2y - z - 3 = 0 \;\&\; 3x - y + 2z - 1 = 0$

$$\therefore \begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda - 5 = 0$

$\Rightarrow \lambda = 5$

$\therefore eq.\ of\ \pi : 7x - 7y + 8z + 3 = 0$

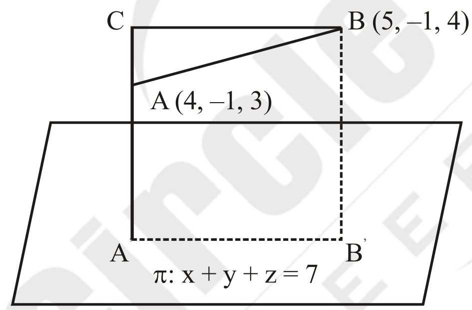
Distance of π from origin $= \left| \frac{3}{\sqrt{162}} \right| = \frac{1}{3\sqrt{2}}$

84. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\sqrt{\frac{2}{3}}$

Solution: (4)



Projection of B on A = AC

$AC = \overrightarrow{AB} \cdot \widehat{AC}$

Now $\overrightarrow{AB} = \hat{i} + 0\hat{j} + \hat{k}$

$\overrightarrow{AC} = \hat{i} + \hat{j} + \hat{k} \qquad \left[\because \overrightarrow{AC} \perp \pi\right]$

$\Rightarrow AC = \frac{(\hat{i} + 0\hat{j} + \hat{k})(\hat{i} + \hat{j} + \hat{k})}{\sqrt{1^2 + 1^2 + 1^2}}$

$AC = \frac{2}{\sqrt{3}}$

Now in rt. ΔABC,

$AC^2 + BC^2 = AB^2$

$\Rightarrow BC^2 = AB^2 - AC^2 \; = 2 - \frac{4}{3}$

$\Rightarrow BC = \sqrt{\frac{2}{3}} \qquad \Rightarrow A'B' = \sqrt{\frac{2}{3}}$

85. Let $\vec{u}$ be a vector coplaner with the vector $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to :

(1) 336 (2) 315

(3) 256 (4) 84

Solution: (1)

$$\vec{U}=\lambda\left(\vec{a}\times\vec{b}\right)\times\vec{a}$$

$$=\lambda\left\{\left|\vec{a}\right|^2\cdot\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{a}\right\}$$

$$=\lambda\left\{-4\hat{i}+8\hat{j}+16\hat{k}\right\}$$

$$\vec{u}=\lambda'\left(-\hat{i}+2\hat{j}+4\hat{k}\right)$$

$$\Rightarrow\vec{u}\cdot\vec{b}=24$$

$$\Rightarrow\lambda'=4$$

$$\Rightarrow\vec{u}=-4\hat{i}+8\hat{j}+16\hat{k}$$

$$\Rightarrow\left|\vec{u}\right|^2=336$$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two addtional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{3}{10}$ (2) $\frac{2}{5}$

(3) $\frac{1}{5}$ (4) $\frac{3}{4}$

Solution: (2)

Let E_1 be the event a red ball is drawn

E_2 be the event a black ball is drawn

A be the eent 2nd drawn ball is red.

$$\Rightarrow P(A)=P(E_1),P(A/E_1)+P(E_2),P(A/E_2)$$

Now $\Rightarrow P(E_1)=\frac{4}{10}\ \frac{2}{5}$ $\quad P(E_2)=1-\frac{2}{5}=3/5$

$$P(A/E_1)=\frac{6}{12}\ \frac{1}{2}\qquad P(A/E_2)=\frac{4}{12}\ \frac{1}{3}$$

$$P(A)=\frac{2}{5}\times\frac{1}{2}+\frac{3}{5}\times\frac{1}{5}$$

$$=\frac{1}{5}+\frac{1}{5}=\frac{2}{5}$$

87. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is :

(1) 9 (2) 4

(3) 2 (4) 3

Solution: (3)

$$\sum_{i=1}^{9}(x_i-5)=9\ \&\ \sum_{x=1}^{9}(x_i-5)^2=45$$

$$\sigma^2=\frac{\sum_{i=1}^{9}(x_i-5)^2}{N}-\left(\frac{\sum(x_i-5)}{N}\right)^2$$

$$\sigma^2=\frac{45}{9}-\left(\frac{9}{9}\right)^2$$

$$\sigma^2=5-1$$

$$\sigma^2=4$$

$$\sigma=\sqrt{4}$$

$$SD=\sigma=2$$

88. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$

(3) $\frac{8}{9}$ (4) $\frac{20}{9}$

Solution: (2)

$$8\cos x\left[\cos\left(\pi/6+x\right).\cos\left(\pi/6+x\right)-1/2\right]=1$$

$$\Rightarrow 8\cos x\left[\cos^2\pi/6-\sin^2x-1/2\right]=1$$

$$\Rightarrow 8\cos x\left[\frac{3}{4}-\sin^2 x-\frac{1}{2}\right]=1$$

$$\Rightarrow 8\cos x\left[\frac{1}{4}-\sin^2 x\right]=1$$

$$\Rightarrow 2\cos x-8\cos x\cdot\sin^2 x-1=0$$

$$\Rightarrow 2\cos x-8\cos x\left(1-\cos^2 x\right)-1=0$$

$$\Rightarrow 8\cos^3 x-6\cos x-1=0$$

$$2\left[4\cos^3 x-3\cos x\right]=1$$

$$2\cos 3x=1$$

$$\cos 3x=\frac{1}{2}$$

$$3x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3}$$

$$x=\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9}$$

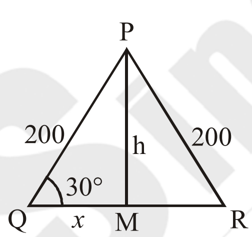
$$\Rightarrow k=\frac{1}{9}+\frac{5}{9}+\frac{7}{9};\quad k=\frac{13}{9}$$

89. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) 100 (2) 50

(3) $100\sqrt{3}$ (4) $50\sqrt{2}$

Solution: (1)



Let $QM = x \Rightarrow RM = x$

Let height of tower = h

Now $\because$ angle of elevation at P is 45°

$$\Rightarrow \frac{h}{PM}=\tan 45^\circ$$

$$\Rightarrow PM = h$$

Also angle of elevation at Q is 30°

$$\Rightarrow \frac{h}{QM}=\tan 30^\circ$$

$$\Rightarrow \frac{h}{x}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow x=\sqrt{3}h$$

Now in rt ΔPQM

$$\Rightarrow PM^2+QM^2=PQ^2$$

$$\Rightarrow h^2+\left(\sqrt{3}h\right)^2=(200)^2$$

$$\Rightarrow 4h^2=200\times 200$$

$$\Rightarrow h^2=50\times 200$$

$$\Rightarrow h=100m$$

90. The Boolean expression ~(p∨q) ∨ (~p∧q) is equivalent to :

(1) ~ p (2) p

(3) q (4) ~ q

Solution: (1)

$$\sim(p\vee q)\vee(\sim p\wedge q)$$

$$=(\sim p\wedge \sim q)\vee(\sim p\wedge q)$$

$$=(\sim p)\wedge(\sim q\vee q)$$

$$=(\sim p)\wedge t$$

$$=\sim p$$